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Long-Range Interactions of the Ball

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Abstract

In model independent way considered long-range interactions of the ball under assumption that any particle of this objects interact with probe particle as A/r^n. Also presented model-independent corrections to the Coulomb energy levels from regularised version of the potential A/r^n.

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In this article considered long-range interactions of the ball under assumption that the unit of the its volume interact with probe particle as A/r^n. There are many examples of the such singular potentials: e.g. Van-der-Vaals interactions. Also must be noticed that in some of the theories with higher dimensions (see e.g. [1],[2] and references therein) appear the following corrections to Newton potential [3]:

$$U(r) = \frac{Gm_1m_2}{r}(1 + A/r^n) \tag{1}$$

Previously has been considered the following corrections to Newton potential which also appear in some of multidimensional theories (see references in [4]):

$$U(r) = \frac{Gm_1m_2}{r}(1 + se^{-Mr}) \tag{2}$$

In number of papers (see references in [4]) has been calculated the potential of the ball with homogeneous density under assumption that any particle of the ball interact with probe particle in accordance with formula (2). In [4] has been also published the experimental restriction on parameters s and$M$. Analogously, using data of the experiments described in [4] in principle possible to obtain some restriction on A and n.

We present also model-independent corrections to the energy levels of the electron in the Coulomb field of the nuclei from regularized version of the potential A/r^n.

We obtain for potential of ball with constant density :

$$U_n(r) = \int d^3r_1 \frac{A}{|\vec{r} - \vec{r'}|^n} = -\frac{2\pi A}{r}\int r'dr'(\frac{|r-r'|^{(2-n)}}{2-n} - \frac{(r+r')^{(2-n)}}{2-n}) \tag{3}$$

At $r > R$ we have after integration:

$$U_n(r) = \frac{2\pi A}{r}\frac{1}{n-2}[\frac{(r-R)^{4-n}}{4-n} - \frac{r(r-R)^{3-n}}{3-n} - \frac{(r+R)^{4-n}}{4-n} + \frac{r(r+R)^{3-n}}{3-n}] \tag{4}$$

From (17) it is seen that potential is singular at $r \to R$ ($U(r) \sim |r-R|^{(4-n)}$.

From this formula it is seen that cases $n = 2, 3, 4$ must be considered separately.We obtain:

$$U_2(r) = \frac{2\pi A}{r}(rR + \frac{1}{2}(R^2 - r^2)log(\frac{r+R}{|r-R|}), \tag{5}$$

$$U_3(r) = \frac{2\pi A}{r}[rlog(\frac{r+R}{r-R}) - 2R], \tag{6}$$

$$U_4(r) = \frac{\pi A}{r}[log(\frac{(r-R)}{r+R}) + \frac{2rR}{r^2 - R^2}]. \tag{7}$$

During derivation of $U_2(r)$ has been taken into account potential $U_n(r)$ at $r < R$ (see formula (11) below). From (4) for $n = 5, 6, 7$ we obtain the following simplifications:

$$U_5(r) = \frac{4\pi AR^3}{3r}\frac{1}{(r^2 - R^2)^2} \tag{8}$$

$$U_6(r) = \frac{4\pi AR^3}{3}\frac{1}{(r^2 - R^2)^3} \tag{9}$$

$$U_7(r) = \frac{4\pi AR^3}{15r}\frac{(5r^2 + R^2)}{(r^2 - R^2)^4} \tag{10}$$

At $n = 1$ we obtain as it must be $U_1(r) = 4\pi R^3 A/3r$. At $r < R$ and $n < 3$ we have:

$$U_n(r) = \frac{2\pi A}{r}\frac{1}{n-2}[\frac{(R-r)^{4-n}}{4-n} + \frac{r(R-r)^{3-n}}{3-n} - \frac{(r+R)^{4-n}}{4-n} + \frac{r(r+R)^{3-n}}{3-n}] \tag{11}$$

At $n \geq 3$ integral $\int r'dr'\frac{1}{(r-r')^{(2-n)}}$ is divergent if $r < R$.

It mean that inside ball ($r < R$) and $n \geq 3$ we must regularised interaction A/r^n. We choose the following regularizations:

$$V_n(r) = \frac{A}{(r^2 + a^2)^{n/2}} \tag{12}$$

$$V_n(r) = \frac{A}{(r+a)^n}, \tag{13}$$

$$V_n(r) = \frac{A}{r(r+r_0)^n}. \tag{14}$$

where$r_0, a << R$. For example a may be a size of the molecules (atoms) which contained in this ball.The last potential transformed into Coulomb potential at $r << r_0$.

Using regularized potential (12) we obtain for $n = 5$ the following result:

$$V_5(r) = \frac{2\pi A}{5r}[-f_- + f_+ + \frac{r}{a^2}((R-r)f_- + (R+r)f_+)] \tag{15}$$

where $f_\pm = ((R \pm r)^2 + a^2)^{-\frac{1}{2}}$ At $r > R$ and $|r - R| >> a$ we again obtain A/r^n, and at $r < R$,$r, |r - R| >> a$ we have:

$$V_5(r) = \frac{4\pi A}{5}[\frac{1}{a^2} - \frac{1}{R^2 - r^2}]. \tag{16}$$

Now we consider in model independent way (the nature of correction may be any including cases which has been considered above) corrections to the energy levels of the electron in the Coulomb field of the nuclei from regularised version of the potential A/r^n expressed by formula (14):

$$\delta E = \int d^3r|\psi|^2 A/(r(r+r_0)^n) \approx 4Aa_B^{-3}r_0^{2-n}\frac{2n-3}{(n-2)(n-1)} \tag{17}$$

Here has been taken into account that at $r_0 << a_B$ in integral we can put $\psi^2 = 1/\pi a_B^3$ at $l = 0$. Analogously, in case of potential (13) we obtain:

$$\delta E = \int d^3r|\psi|^2 A/((r+r_0)^n) \approx 8Aa_B^{-3}r_0^{3-n}\frac{1}{(n-3)(n-2)(n-1)}. \tag{18}$$

This formulas are valid if $n > 3$. At $n = 3$ by consideration of two range in integral ($0 < r < L$, $L < r < \infty$ where $r_0 << L << a_B$) e.g. from (22) we

obtain:

$$\delta E = \int d^3r|\psi|^2 A/((r+r_0)^3) \approx 4\pi A(1/3 - 2 + log(a_B/2r_0) - C) \quad (19)$$

where C-is Eiler constant.

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